Modified Newton Kantorovich Methods for Solving Microwave Inverse Scattering Problems

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ABSTRACT

The Modified Newton-Kantorovich method (MNK) was formulated due to the limitation of The Newton-Kantorovich method (NK) in reconstructing the imitation of bone muscle and fat object. It was sensitive to contrast and cell size. In this research MNK and NK methods were applied to reconstruct the dielectric properties distribution of homogeneous and inhomogeneous objects from simulated scattered field dataset to know how the results of image reconstruction using both methods were. The results revealed that the MNK method was more flexible than the NK one.

Keywords: Modified Newton-Kantorovich, microwave inverse

INTRODUCTION

Microwave Tomography imaging can be produced by illuminating a dielectric body with a microwave field in different directions, and measuring the scattered field around the object at each illumination. The data of the measured scattered fields are processed using image reconstruction algorithms to produce the electric property distribution of the object. Generally, the effect of diffraction could not be neglected (Nugroho 2004) thereto the images are difficult to be reconstructed due to the non-linearity and ill-possesses of the inverse problem.

The use of the Newton Kantorovich method (NK) for reconstructing conductive cylinders perfectly from TE scattering was proposed by Roger (Roger 1981). The NK is the generalization of Newton method in which the gradient of the function is defined by Frechet differential of linear function. Accordingly, the update difference in each iteration is the inverse of the difference between the measured values and the update values. The equation, which is elementary in the Newton algorithm, then becomes a functional equation. Furthermore, the inverse problem by NK methods needs to be regulated by Tikhonov-Miller method (Tikhonov 1977). The numerical simulation showed that the illuminated face of the cylinder could still be well reconstructed when the wavelength decreased. Roger's success in applying NK for TE electromagnetic inverse scattering had made this method easier applied to the TM scattering from a cylinder.

Joachimowicz (1991) demonstrated the advantages of the NK method in reconstructing the complex permittivity of an object when illuminated with electromagnetic waves. The scattered fields were first obtained from the initial guesses ξ_0 values of the contras dielectric properties. The experimental arrangement was set in 1998 (Joachimowicz 1998). The results show that NK method was flexible in experimental arrangement, choice of polarization and prior knowledge. The NK method could determine the shape and location of an object well for noiseless data (Belkebir 1997), but it failed to converge for noisy data for SNR 30dB (Nugroho 2003). The NK method was faster and more flexible than various gradient methods (Nugroho 1999) for low contrast objects but the method was sensitive to high contrast object. Therefore the sensitivity of the method is not suitable for industrial application.

New methods called Modified NK method is proposed in this work to improve the sensitivity of the NK method. The MNK method is the NK algorithm with improved approximation of internal fields. How do they diver in the result of reconstruction. The MNK is investigated and compared in term of their sensitivities to dielectric contrast, cell size, and noise level.

The NK method in microwave imaging application was operated by inverting the data of scattered fields into the distribution of dielectric properties of the object when the object was illuminated by a TM wave. This method starts by initializing the distribution of dielectric constant of the object with guessed values, which could be a constant or contain priori information of the object to be reconstructed. The scattered electric fields at the points of observation are then calculated with the integral equation and numerical approximation. This process is commonly known as forward or direct problem. Then, the differences between measured scattered fields and the calculated scattered field with multiple observation points and multiple EM incidences are inverted to obtain the updated values of dielectric distribution. This was repeated until the differences between the measured and calculated scattered fields are smaller than the limit of convergence

Let us consider an inhomogeneous 2D dielectric object, with a cross section described in x_0y plane coordinates. The cross section D was divided into N sufficient cells so that the electric field and the dielectric constant can be assumed to be constant over each cell. Denote E_n and ξ_n are the electric field and the dielectric constant contrast of n^th^ cell, then the total field, every where inside and outside the object, satisfied the equation

$$E_n = E_I - \sum_{n'=1}^{N} C_{nn'} \xi_{n'} E_{n'} \tag{1}$$

Where

If $n \neq n'$

$$C_{nn'} = \tfrac{1}{2} j\pi k_0 a_{n'} J_1(k_0 a_{n'}) H_0^{(2)}(k_0 \rho_{nn'})$$

If

$n = n'$

$$C_{nn'} = \tfrac{1}{2} j\pi \left[k_0 a_{n'} H_0^{(2)}(k_0 \rho_{nn'}) - \tfrac{1}{\pi} 2j \right]$$

Similarly, the relation between the total field at the m^{th} point outside object domain and the total field in the M number of point outside is

$$E_m = E_I - \sum_{n'=1}^{N} C_{mn'} \xi_{n'} E_{n'} \tag{2}$$

where

$$C_{mn} = \tfrac{1}{2} j\pi k_0 a_n J(k_0 a_n) H_0^{(2)}(k_0 \rho_{mn})$$

Thus the scattered field at m^{th} point can be written as

$$E_{Sm} = -\sum_{n'=1}^{N} C_{mn'} \xi_{n'} E_{n'} \tag{3}$$

where m=1, 2 .. , M

In the small variation Δ of the scattering field E_S and the total field E and constant Incident Field, the (3) can be transformed into.

$$\mathbf{\Delta E} = -\mathbf{C}_{nn'} \mathbf{\Delta}(\xi \mathbf{E}) \quad \text{and}$$

$$\mathbf{\Delta E}_s = -\mathbf{C}_{mn'} \mathbf{\Delta}(\xi \mathbf{E}) \tag{4}$$

Substituting the first-order-approximation of $\Delta\xi\mathbf{E}$, which is $\Delta\xi E + \xi\Delta E$, into (4) lead to the NK method expression

$$\mathbf{\Delta E}_s = \mathbf{D}\mathbf{\Delta}\xi \tag{5}$$

Where

$$\mathbf{D} = -\mathbf{C}_{mn'} \left[\mathbf{I} + \xi \mathbf{C}_{nn'} \right]^{-1} \mathbf{E}_{n'}$$

It can be seen that **D** is a function of distribution $\mathbf{E}_{n'}$ and **ξ. E** everywhere inside the object can be obtained from equation

$$\mathbf{E} = \left[\mathbf{I} + \xi \mathbf{C}_{nn'} \right]^{-1} \mathbf{E}_I \tag{6}$$

The inversion above was always made since the matrix to be inverted was square but normally its condition number was relatively small. When examined closely, this equation has a shortcoming as it has element variable ξ and E_I. If the value of ξ relatively close to its actual value then **E** were also approach the actual value and **D** would approach the expected differential operator of the direct problem. However, if value ξ were relatively far from the actual value the **D** would act as error operator. A new method is therefore proposed to estimate better **E** by applying NK method to equation 6, which lead to

$$\mathbf{\Delta}(\xi \mathbf{E}) = \left[\alpha \mathbf{I} + \mathbf{C}^{\#}_{mn'} \mathbf{C}_{mn'} \right]^{-1} \mathbf{C}^{\#}_{mn'} \mathbf{\Delta E_S} \tag{7}$$

It was essential to multiply $\mathbf{C}_{mn'}$ with its conjugate transpose due to $\mathbf{C}_{mn'}$ not to be square. Since the number of rows was larger than the columns, squaring the matrix has led to a bad condition. Consequently, a regulator α was needed. The use of α and the search for it was similar to the original NK method (Nugroho 1999). Finally, substituting 6 into 7 results the update value for **E** inside the object.

$$\mathbf{\Delta E} = -\mathbf{C}_{\mathbf{nn'}} \left[\alpha \mathbf{I} + \mathbf{C}^{\#}_{mn'} \mathbf{C}_{mn'} \right]^{-1} \mathbf{C}^{\#}_{mn'} \mathbf{\Delta E_S} \tag{8}$$

Once better values of **E** are obtained, the values of **D** can also be better approximated and **D** will be close to the differential operator in the equation of the direct problem.

METHODS

The MNK and NK were applied to reconstruct some configured objects. The qualities of these two algorithms were studied through the effects of 1) the contrast of the object, and 2) cell size. Based on the numerical results, the algorithms will be compared in term of maximum capability to the level of the effects to be tested, and the needed time of algorithms to produce image with a 10% relative error.

Homogeneous and inhomogeneous objects of circular shape were employed (see Figure 1). The homogenous object had equal contrast permittivity in every cell. The inhomogeneous object had two different permittivity. The cross section on the circular object is divided into 196 cells. Each cell has the same area as a small circle radius

$$r_c = \frac{r_o}{N_r \sqrt{m_o}} \qquad (10)$$

where

r_c is the radius of small circle

r_o is the radius of circular object

N_r is the number of ring inside the object

m_o is the number of cell in the innermost part of the ring.

To measure the quality of an algorithm, the relative error of the image as a result of reconstruction is determined by,

$$err\xi = \frac{\left\| \xi_{k+1} - \xi \right\|}{\left\| \xi \right\|} 100\%$$

The needed time of the algorithm in producing an image with relative error equal to 10% was determined using the counter of time in the algorithm. In addition the maximum number of iteration (MaxIt) was set to 50 in every test. When MaxIt was reached but the relative error was still above 10%, the approximated time needed to reach 10% of error was determined using

$$t_{10\%} = \frac{\left[t_k\left(err\xi_{k-n} - 10\%\right)\right] - \left[t_{k-n}\left(err\xi_k - 10\%\right)\right]}{err\xi_{k-n} - err\xi_k}$$

Where

$t_{10\%}$ is time needed to reach 10% error

t_k is time needed to reach k^{th} iteration

t_{k-n} is time needed to reach $(k\text{-}n)^{th}$ iteration

n is a small integer number

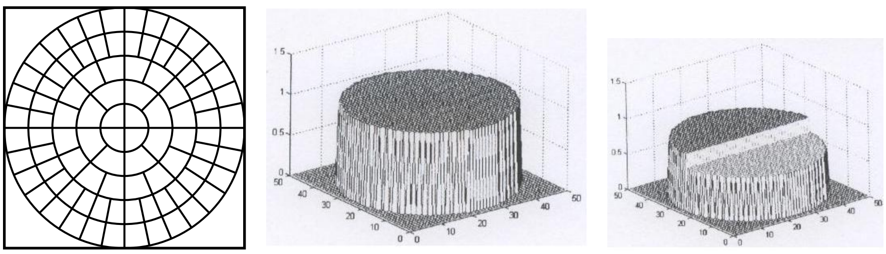
The effect of contrast of the object on images reconstructed was studied by considering the needed time of algorithms to produce image with 10% error. In this study, the object size was fixed and made at 2.454 GHz for comparison purposes. The dielectric contrast of the object was varied for each reconstruction. For homogeneous object the contrast was set 0.1 to 2.5. For inhomogeneous object, the first half of the objects was set with 0.6 contrasts and the other half was set various between 0.0 to 2.0.

The effect of the cell size was studied by varying the object size while the number of cell kept constant. By doing this the cell size will vary. On each variation the algorithms are applied to reconstruct the two objects.

RESULTS AND DISCUSSION

Study of the contrast of the object

In the inverse scattering problem, the contras distribution of the object was the weight vector of the problem. This vector has to be solved to minimize the error of the problem. In the case of single layer quadratic function of weight, the weight vector that minimizes the function was single but the error surface is generally a multidimensional parabolic form.



(a) (b) (c)

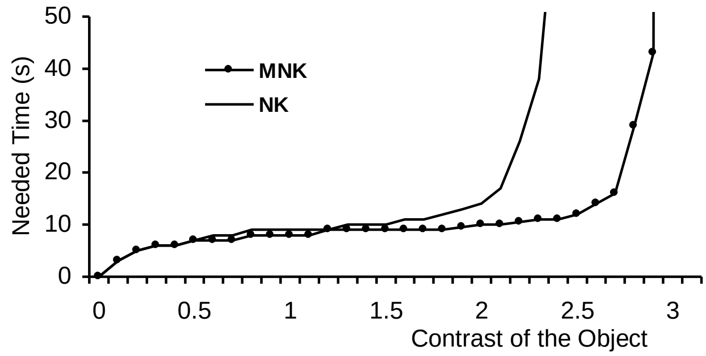
Figure 1. The object used in the test. a) the cross section of the circular object. b) The original homogeneous real image with ξ=1. The diameter of the object is 0.66λ. the contrast c) The real part of the inhomogeneous object. The top half contrast is 0.6 and the other half is 0.4. The imaginary part of all objects is set to be 0.

Furthermore the inverse scattering problem has more than one layers of adaptive weight, thus the error function were typically formed by a highly non linear function of weights, and there may exist many minima all of which the weight vector satisfies the minimum condition.

As a result of numerical simulation, the needed time by those two algorithms to reconstruct good image against the dielectric contrast of the object was shown in figure 2. The images produced by the reconstruction of the homogeneous circular object after 200-second of process are shown in Figure 3. In general the contrast will influence the needed time of the algorithms to create image with 10% error. The larger the contrast differences the longer the needed time would be. This was so because of the distance between guessed vector and the exact weight vector. Increasing the distance will make the searching likely to be trapped at local minimum point or another false points that fulfills the algorithms condition, but does not meet the condition of the problem. This can be seen in Figure 1, the error revolution upon the iteration appears to be slow as the contrast increases. If the contrast was above the edge of the limit the error will never reach 10%; thus the needed time to produce good image was infinitive.

The result shows that the needed time of reconstruction to produce good image increases as the contrast of the object increases. The needed time of MNK method was less sensitive to high contrast object compared to the NK method. When the object contrast was less than 0.5 for homogeneous object and 1.3 for inhomogeneous object, both MNK and NK methods have approximately equal needed time to obtain good image. However, when the contrast of the object was higher the needed time of MNK was smaller than NK. The needed time of the NK method increases sharply when the contrast reaches 2.1 and 1.7 for homogeneous and inhomogeneous object respectively. On the other hand the MNK remains steady until 2.9 and 2.4. The result means that MNK was more flexible to high contrast compared to NK method.

(a)



(b)

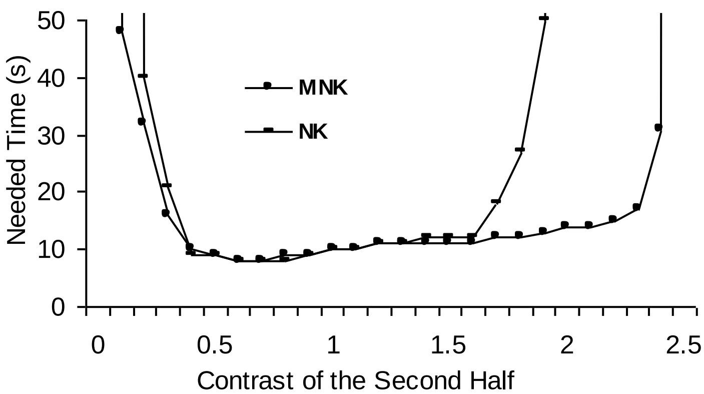
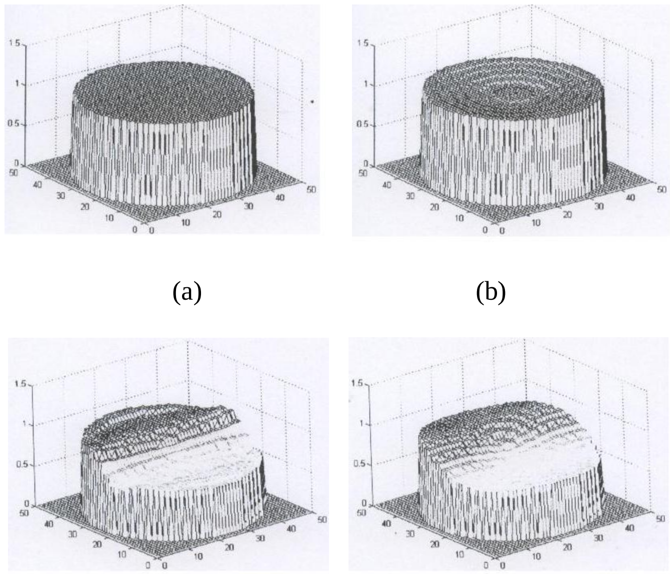


Figure 2. The time needed by the MNK and NK methods to obtain good a) homogeneous image and b) inhomogeneous image.



(a) (b)

(c) (d)

Figure 3. The images produced by the MNK method (a) and (b), and by the NK method (c) and (d) when reconstructing homogeneous objects (a) and (b) and inhomogeneous objects (c) and (d) .

Study of the cell size of the object

In microwave scattering, cell size was one of the variables in discretised integral equations. In the approximation proposed by Richmond (Richmond 1965), the formulation of scattered fields depends on the cell size and wave number. Therefore, there should be a compromise between the cell size and the wave number in order to keep the approximation in good order. Joachimovich (1998) suggested a compromise value for the NK method, which was 0.1λ for the fine mesh, and 0.3λ for coarse mesh. Apart from this, the effect of the cell size of the process of reconstruction has not been reported in the literature.

As shown in Figure 4 the needed time of the algorithms to produce good image increases as the cell size increases. The needed time of MNK was smaller than NK method; this means that the MNK method was capable of improving the speed of the NK method. When the radius of the cell is less than 0.02λ, both methods have approximately the same needed time.

However, when the radius of the cell was more than 0.02λ, the needed time of MNK method was smaller than the needed time of the NK method. As it was observed with numerically simulated experimental data, the needed time of the algorithm decreases when the mesh size was reduced. The discretisation effect for MNK and NK methods have been assessed by considering fine cell sizes 0.07λ and 0.06λ, respectively. If the cell size was too large compared to the fine sizes, the algorithm will diverge.

Principally, the MNK method improves the internal field estimation of the NK method. The results of simulation show that modification of NK gives a positive effect to the sensitivity of the algorithm. The MNK method gives an increasing of 10% and15% of the cell radius limit for homogeneous and inhomogeneous objects respectively. In the inverse scattering problem, the cell size variation affects the condition of the integral equation. Both forward and inverse problems are function of wave number and the radius of the cells. Therefore, there should be a compromise between the wave number and cell radius so that the condition of the operator was well behaved. However the cell size can be reducing by increasing the number of the cell, this would increase the calculation burden.

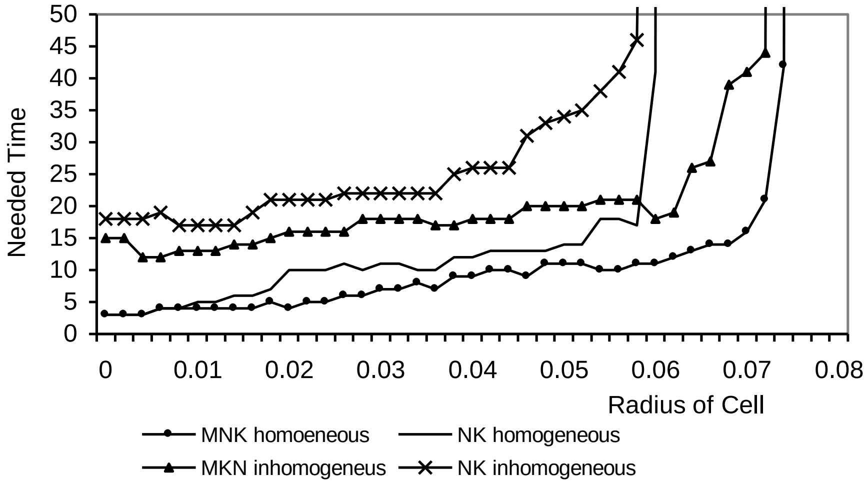
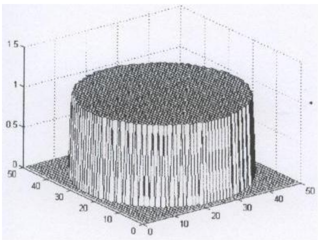
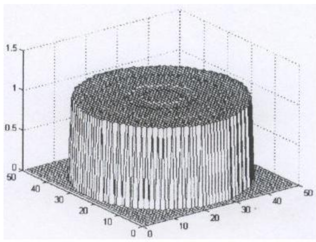
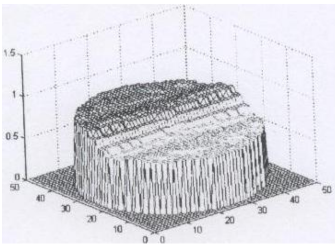
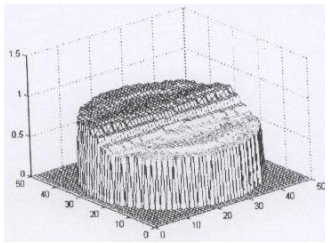


Figure 4. The Needed time for the MNK method and NK method to reconstruct homogeneous and in homogeneous objects in various cell size.

(a) (b)

(c) (d)

Figure 5. The images of the homogeneous and inhomogeneous objects reconstructed with radius of cells = 0.05λ by (a) and (c) MNK and (b) and (d) NK methods after 200 seconds of processes at 2.454GHz.

Table 1. The fine cell of the object reconstructed by the MNK method and the NK method.

Algorithm	Radius of the cells (λ)	
	Homogeneous	Inhomogeneous
MNK	0.074	0.068
NK	0.062	0.058

CONCLUSION

As it has been observed with numerical simulated experimental data, the algorithms are sensitive to the level of object contrast, cell size and noise. The MNK method shows better contrast flexibility compared to the NK method. All algorithms tested gave satisfactory results for low contrast objects, but for high contrast objects the MNK method was less sensitive than the NK method in reconstructing the image. In homogeneous case, the MNK method was faster than the NK method but for inhomogeneous case, they have similar range of capability in speed.

Concerning the experimental aspects, the frequency and cell size affect directly the Green's operator. At the numerical level, it appears that the frequency and cell size have to compromise to maintain converging of the iteration process. From the results of the numerical experiment, it can be concluded that the MNK method was more flexible than NK method for combination of frequency and cell size. Reducing the error of the total internal field can significantly reduce the error on the results. In the MNK method the total internal field was corrected first by using the combination of dielectric property, frequency and cell size before guessing the scattered field. This will allow the MNK to jump from the local solution.

Contribution has been made in proposing the MNK method for solving two-dimensional inverse scattering problem. The MNK was developed by modifying the NK method. Numerical simulations show that MNK method was more flexible to various effects and faster.

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